

# Common Core Mathematics

Preparing Students for College and Career  
Readiness



**LBUSD  
SCHOOL BOARD STUDY SESSION  
MARCH 31, 2014**



# Competitive in a Global Economy

2

As California competes for jobs in an increasingly competitive global economy, the state faces a looming shortage of highly educated workers. For a variety of reasons, the need for the individuals with degrees in science, technology, engineering, and mathematics (STEM) is of particular concern. If we want California students to be able to successfully pursue higher education, especially in STEM, keeping them on track throughout their middle-and high-school academic experience is essential. Nowhere is this more true than in the discipline of mathematics where understanding develops cumulative, requiring that students master progressively more complex building-block concepts and skills in order to be successful in each next-higher-level course."

- [College Bound - How Math Course Sequence Matter, WestEd 2012](#)

# Preparing Students for Success in a Global Economy

3

## 21<sup>st</sup> Century Competencies by Categories

### STEM



### Cognitive Competencies

- Academic conceptual & procedural mastery
- Critical Thinking
- Creativity

### Interpersonal Competencies

- Communication and Collaboration
- Leadership
- Global Awareness

### Intrapersonal Competencies

- Growth mindset
- Learning how to learn
- Intrinsic motivation
- Grit

# A Tale of Two Tasks

4

## Task A

It's the end of the semester, and the clubs at school are recording their profits. The Science Club started out with \$20 and has increased its balance by an average of \$10 per week. The Math Club saved \$5 per week and started out with \$50 at the beginning of the semester.

- Create an equation for each club. Let  $x$  represent the number of weeks and  $y$  represent the balance of the club's account.
- Graph both lines on one set of axes. When do the clubs have the same balance?
- What is the balance at that point?

Source: Dietiker, Kysh, Sallee, and Hoey (2008). *College Preparatory Mathematics Algebra Connections California Edition*.

# A Tale of Two Tasks

5

## Task B

### Group Activity

### The Basic Student Budget

Cal, Bernie, and Doc are college students on budgets.

Sometimes the three have a little difficulty keeping to their budgets. Their biggest problem is the rent.

The total rent for their apartment is \$900, which is split evenly among the three roommates. The rent is due on the last day of each month. The guys don't get paid until the first day of the next month.

Their landlord has no tolerance for late payments.

Each student had a different amount of money after being paid on April 1. At the end of that day, Cal had \$1,100, Bernie had \$800, and Doc had \$600. As the month goes by, they each occasionally note how much they had left at the end of the day.

The table shows their records so far.

Amount of Money Remaining (in dollars)

Date	Cal	Bernie	Doc
April 3	996	766	570
April 10	704	698	490
April 17	440	626	430



**It's April 21, and there's a great concert on campus. This would be an extra cost, beyond the three students' normal expenses. How much, if any, can each one spend and still have enough for rent money on the morning of April 30?**

Adapted from: Fendel, D., Resek, D., Alper, L., and Fraser, S. (2009). *Interactive Mathematics Program Year 1 Second Edition*.

# Developing Students “Habits of Mind”

6

## Rigor in a Global Economy

- Conceptual understanding
- Procedural fluency
- Precision
- Application to new situations

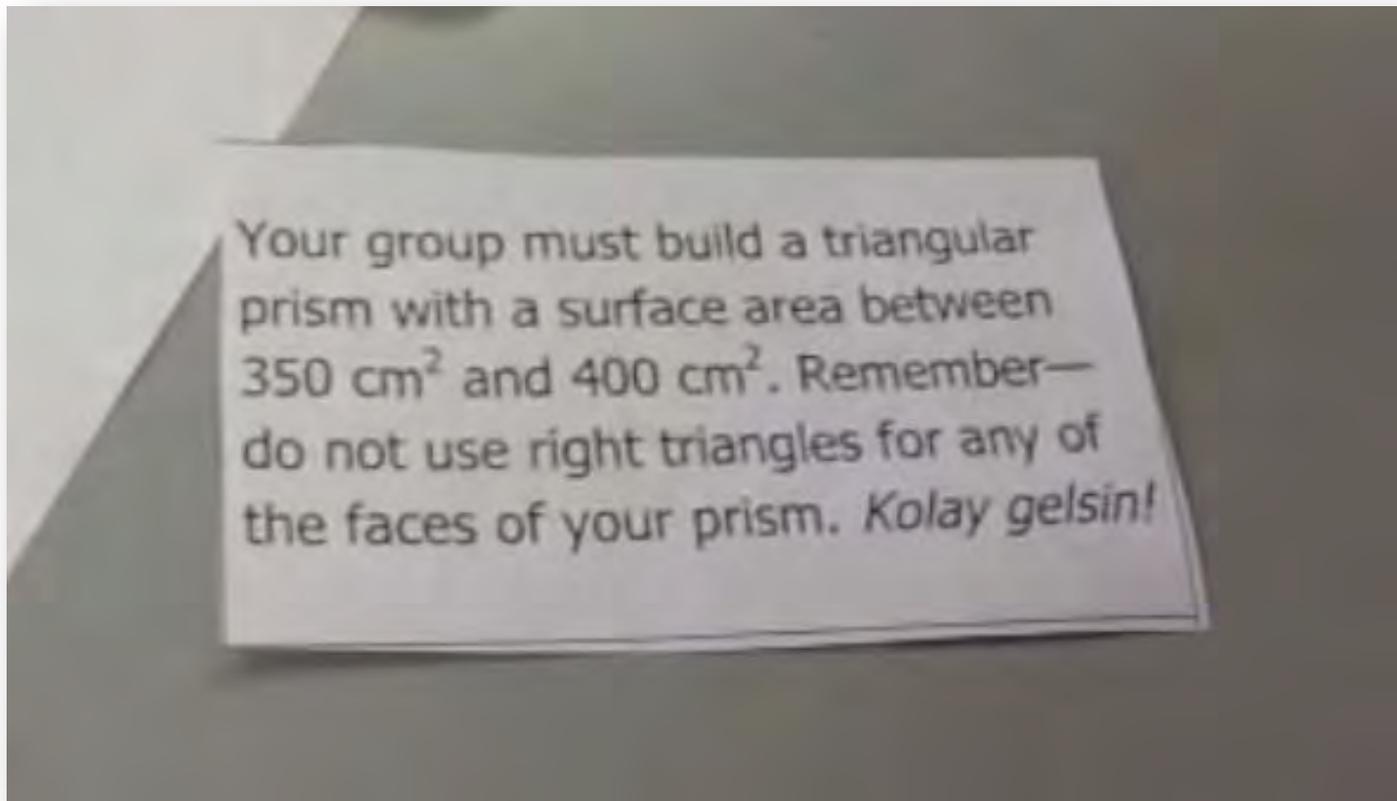
## Math Practice Standards

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

# Developing Students “Habits of Mind”

7

## SMP in Action Video: High School Integrated Math II



# Defining Fluency: 1997 CST Blueprints

8

## Grade 7 Expectations

By the end of grade seven students are **adept at manipulating** and equations and understand the general principles at work. Students **use factoring** of numerators and denominators and properties of exponents. **they compute**

They know the Pythagorean theorem and solve problems in which they compute the **know how to compute**

Students know how to compute the surface area and volume of basic three-dimensional **make conversions** of area and volume change with a change in scale. Students make conversions between different units of measurement. **proficient at changing**

They know and use different representations of fractions **compute** (fractions, decimals, and percents) and a **compute** at changing from one to another.

They increase their facility with ratio and proportion, **graph** compute percents of increase and decrease, and compute simple and compound interest.

They graph linear functions and understand the idea of slope and its relation to ratio.

# Defining Fluency: 1997 CST Blueprints

9

## Algebra 1 Expectations

### **Symbolic reasoning and calculations with symbols**

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

# College Prepared v. College Ready

10

## Preparedness

- College preparatory courses
- AP courses
- Competitive GPA
- Community service
- Extra-curricular activities
- Student leadership



## Readiness

- Comprehend as well as critique
- Read, speak and write academically
- Construct viable arguments
- Value evidence
- Communicate for multiple purposes
- Attend to precision
- Reason abstractly and quantitatively
- Make sense of problems

# Mathematical Proficiency: SBAC

11

**Overall Claim  
Grades 3–8**

- “Students can demonstrate progress toward college and career readiness in mathematics.”

**Overall Claim  
Grade 11**

- “Students can demonstrate college and career readiness in mathematics.”

**Claim #1  
Concepts & Procedures**

- “Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.”

**Claim #2  
Problem Solving**

- “Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.”

**Claim #3  
Communicating Reasoning**

- “Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.”

**Claim #4  
Modeling and Data  
Analysis**

- “Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.”

# Domains and Conceptual Categories

12

K	1	2	3	4	5	6	7	8	HS	
Counting & Cardinality										
Number and Operations in Base Ten						Ratios and Proportional Relationships		Number & Quantity		
			Number and Operations – Fractions			The Number System				
Operations and Algebraic Thinking						Expressions and Equations		Algebra		
								Functions	Functions	
Geometry										
Measurement and Data						Statistics and Probability				Statistics & Probability

Findwell, Bradford & Foughty, Zachary. "Preparing to Implement the Common Core State Standards for Mathematics." Indiana Department of Education and Ohio Department of Education. March 30, 2011

# Preparing for Algebra in Middle School

13

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationships and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Compare numbers	Add and subtract within 20	Use place value understanding and properties of operations to add and subtract	Multiply & divide within 100	Use place value understanding and properties of operations to perform multidigit arithmetic.	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Measure and estimate lengths in standard units	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
Work with numbers 11–19 to gain foundations for place value	Extend the counting sequence	Relate addition and subtraction to length	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities	Represent and analyze quantitative relationships between dependent and independent variables	Use functions to model relationships between quantities
	Understand place value		Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*			
	Use place value understanding and properties of operations to add and subtract		Geometric measurement: understand concepts of area and relate area to multiplication and to addition					
	Measure lengths indirectly and by iterating length units							

<http://achievethecore.org/page/774/focus-by-grade-level>

# Changing expectations

## The trouble with course names

14

In the particular case of mathematics, there is a “vocabulary” around the names of mathematics courses that is likely to cause confusion not only for educators, but also for parents. “Algebra 1” is a course that, prior to CA CCSSM, has been taught in 8<sup>th</sup> grade to an increasing number of students. That same course name will be the default for ninth grade for most students who moving forward will complete the CA CCSSM for grade eight – a course that is more rigorous and more demanding than the earlier versions of “Algebra 1.” Even so, we expect the changes to cause confusion. The single most practical solution is to describe detailed course contents, in addition to course names, as a way of clearing up confusion until “Algebra I” as commonly used, refers to a ninth grade and not an eighth grade course

# CA Framework on Acceleration

15

***Decisions to accelerate students into the Common Core State Standards for higher mathematics before ninth grade should not be rushed.***

- **Placing students into an accelerated pathway too early should be avoided at all costs.**
- It is not recommended to compact the standards before grade seven to ensure that students are developmentally ready for accelerated content.

- CA Framework, Appendix A, page 9

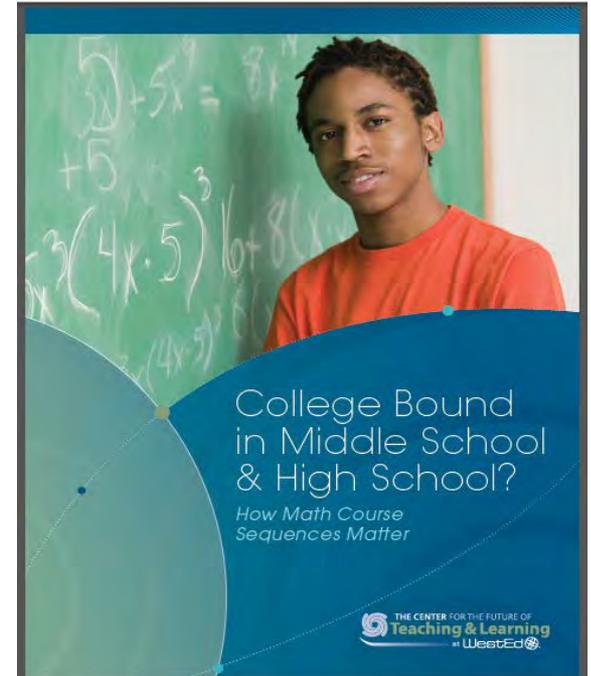
# Changing expectations: Middle School Math

16

When the expectations for middle school mathematics were about speed and accuracy of computations it made sense to accelerate in middle school, and even skip grades.

**This no longer makes sense, the world has changed.**

Middle school mathematics is the key to success for all students. Rushing or skipping is a bad idea for almost all students.



# Research Shows

17

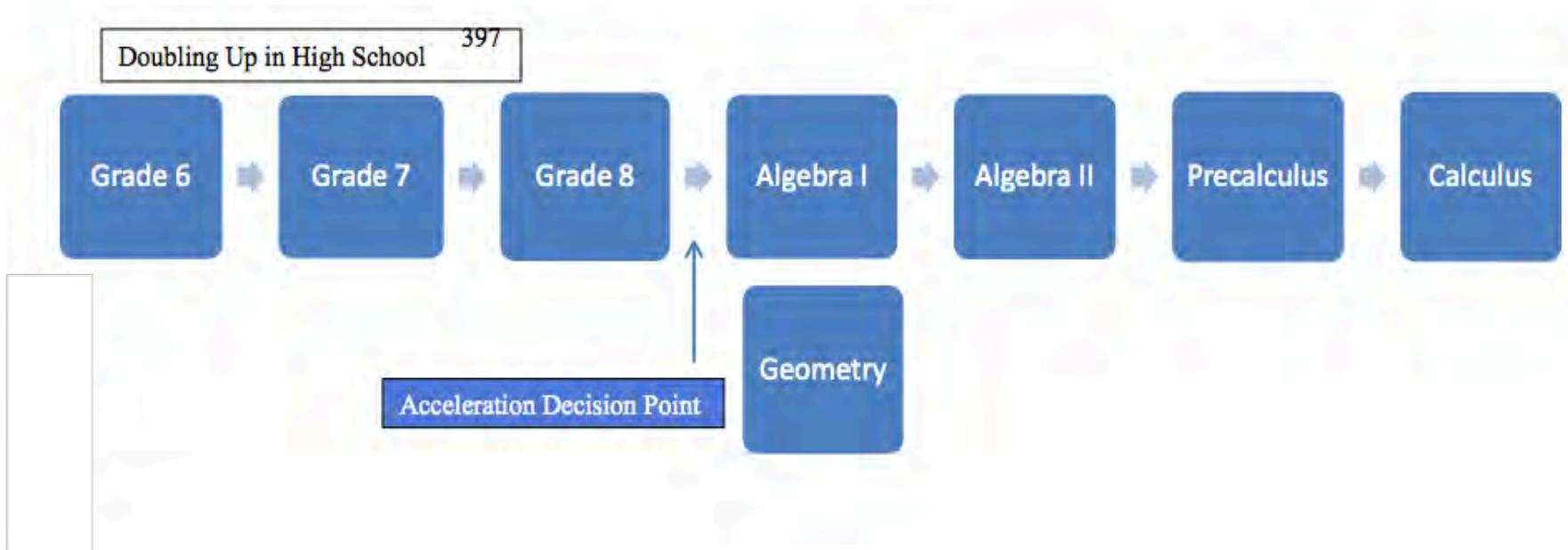
The most important math for 99% of students is middle school math (grade 6 – 8 Common Core Math) – it is the foundation for College and Career Readiness. This especially includes those wanting to go into STEM fields.

“Engineers will be better off with a solid understanding of middle school mathematics”.  
- Patrick Callahan

[http://www.cftl.org/documents/2012/CFTL\\_MathPatterns\\_Main\\_Report.pdf](http://www.cftl.org/documents/2012/CFTL_MathPatterns_Main_Report.pdf)

# Framework Suggested Pathways

18

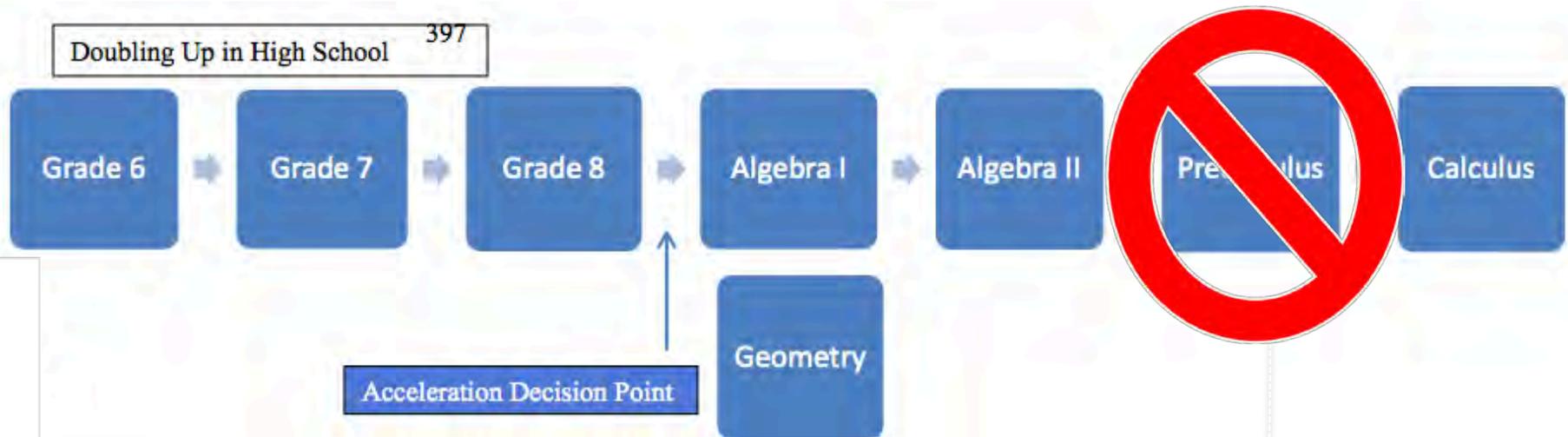


Better than accelerating Middle School.

But doubling up is not necessary!

# Framework Suggested Pathways

19

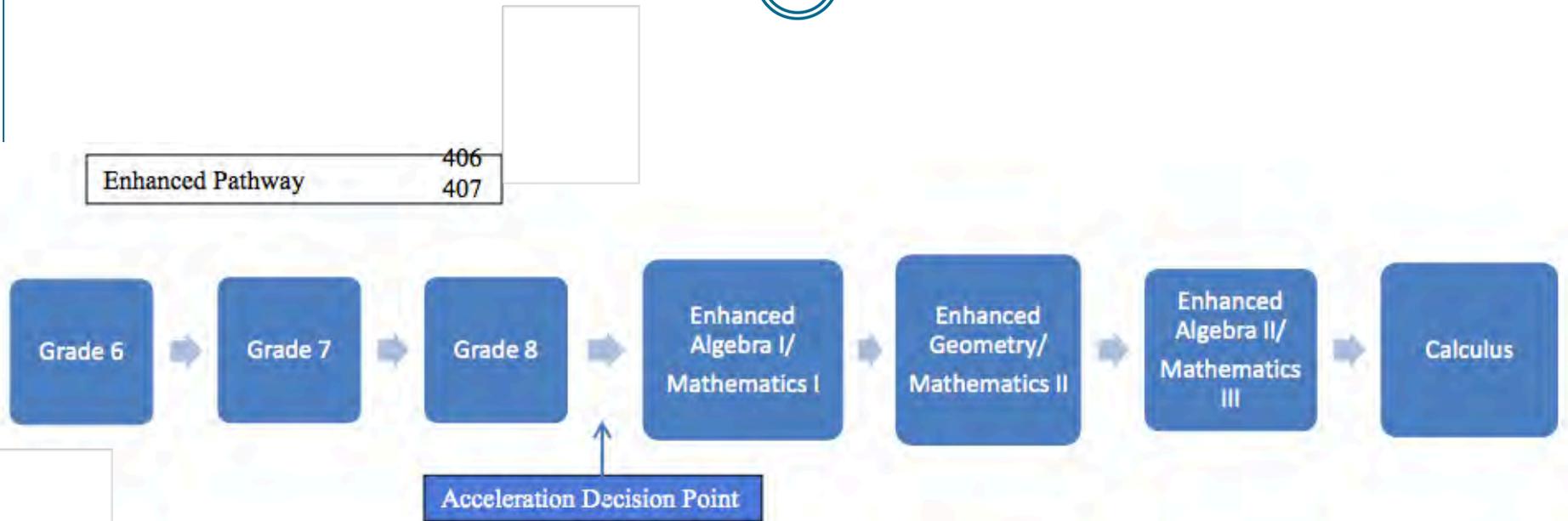


Better than accelerating Middle School.

But doubling up is not necessary! **“Pre-calculus” is not necessary!**

# A better pathway: STEM

20



Enhanced means: Include the (+) pre-calculus standards, go deeper, more rigorous, not skim faster!

## California Mathematics Framework: Possible Pathways to Calculus in 12<sup>th</sup> Grade

For clarity, "HS Course 1, 2 or 3" could refer to either the "traditional" high school pathway (Algebra 1, Geometry, Algebra 2) or "integrated" (Math 1, Math 2, Math 3).

### 1. Compacting in Middle School: Three CCSS courses in two years during grades 7 and 8

Decision point to accelerate: after grade 6

21

Grade 6 students	Grade 7 students	Grade 8 students	Grade 9 students	Grade 10 students	Grade 11 students	Grade 12 students
CCSS 6	CCSS 7 and CCSS 8, part 1	CCSS 8, part 2 and CCSS HS Course 1	CCSS HS Course 2	CCSS HS Course 3	CCSS HS Course 4	AP Calculus

### 2. Doubling Up in High School: Two CCSS courses during two class periods of math in grade 9

Decision point to accelerate: after grade 8

Grade 6 students	Grade 7 students	Grade 8 students	Grade 9 students	Grade 10 students	Grade 11 students	Grade 12 students
CCSS 6	CCSS 7	CCSS 8	1 <sup>st</sup> semester: CCSS HS Course 1	CCSS HS Course 3	CCSS HS Course 4	AP Calculus
			2 <sup>nd</sup> semester: CCSS HS Course 2			

### 3. Compacting in High School: Three CCSS courses in two years during grades 9 and 10

Decision point to accelerate: after grade 8

Grade 6 students	Grade 7 students	Grade 8 students	Grade 9 students	Grade 10 students	Grade 11 students	Grade 12 students
CCSS 6	CCSS 7	CCSS 8	CCSS HS Course 1 and CCSS HS Course 2A	CCSS HS Course 2B and CCSS HS Course 3	CCSS HS Course 4	AP Calculus

### 4. Enhanced Pathway in High School: STEM High School Courses 1, 2, and 3 will include the advanced CCSS (+) pre-calculus standards

Decision point to accelerate: after grade 8 (STEM – Science, Technology, Engineering, and Mathematics)

Grade 6 students	Grade 7 students	Grade 8 students	Grade 9 students	Grade 10 students	Grade 11 students	Grade 12 students
CCSS 6	CCSS 7	CCSS 8	Enhanced (STEM) CCSS HS Course 1	Enhanced (STEM) CCSS HS Course 2	Enhanced (STEM) CCSS HS Course 3	AP Calculus

### 5. Pre-Calculus Summer Bridge Pathway: After completing Courses 1, 2 and 3, students can take a summer course in preparation for Calculus

Decision point to accelerate: after grade 11

Grade 6 students	Grade 7 students	Grade 8 students	Grade 9 students	Grade 10 students	Grade 11 students	Grade 12 students
CCSS 6	CCSS 7	CCSS 8	CCSS HS Course 1	CCSS HS Course 2	CCSS HS Course 3	AP Calculus

Summer  
Pre-Calculus

# Fourth Year Options

22

“The research is clear on the **benefit of students engaging in mathematics throughout all four years of high school**-but that does not mean all students need to, or should, take precalculus or calculus while in high school.<sup>6</sup> Rather, states, districts and schools need to ensure that they are offering **courses that include rich and meaningful mathematics**-whether in traditional mathematics courses, capstone experiences or applied/technical courses with rigorous (and identified) embedded mathematics-particularly for students who complete CCSS-aligned coursework in 10<sup>th</sup> or 11<sup>th</sup> grade.”

Source: 2013 Closing the Expectations gap

<http://www.achieve.org/ClosingtheExpectationsGap2013>

Patrick Callahan stated, “More students would benefit from AP Statistics than AP Calculus” .

- Data analysis is critical for career readiness.

“Consistent with past policy and practice for course approval, **BOARS reiterates its support for either the integrated pathways or the traditional pathways**, as stated in the A-G Guide’s section on Mathematics (“c”). It is BOARS’ expectation that courses developed in accordance with either sequence will receive subject area “c” approval provided that they satisfy the course requirements for area “c” presented in the A-G Guide and that they support students in achieving the Standards of Mathematical Practice given in the CCSSM.”

-April 2013

# Developing Students “Habits of Mind”

24

## Rigor in a Global Economy

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- Procedural fluency
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- Application to new situations

## Math Practice Standards

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# Grades K-8 Focus Areas

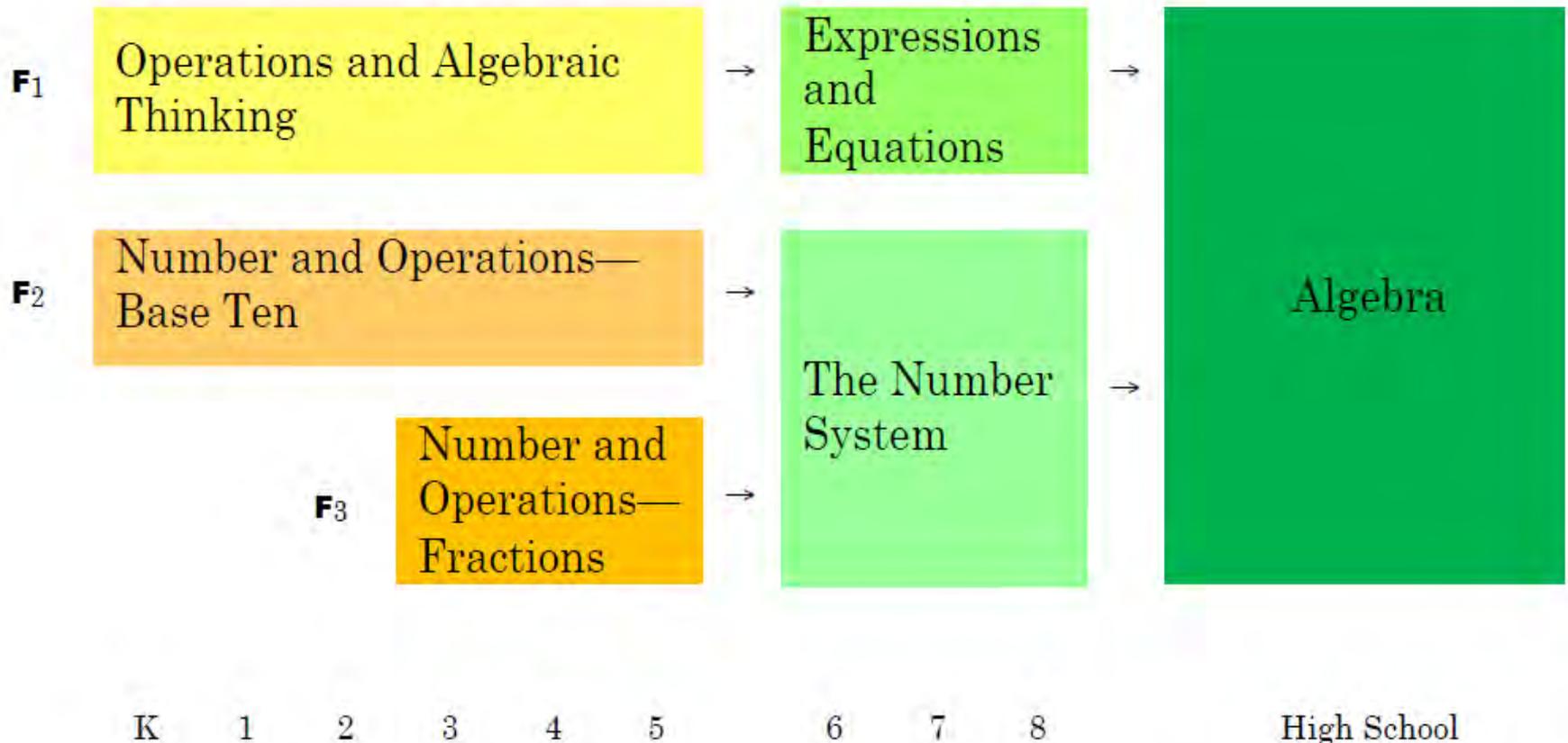
25

K-2	Addition and subtraction concepts, skills, and problem solving, and place value
3-5	Multiplication and division of whole numbers and fractions concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra & linear functions

Source: [www.achievethecore.org](http://www.achievethecore.org)

# Coherence: Flows in the standards

26



Source: William McCallum: "The Common Core: Where do we go from here?"

NCTM 2011 <http://www.nctm.org/conferences/content.aspx?id=29005>

# Rigor: In Major Topics, Pursue Conceptual Understanding, Procedural Skill & Fluency, & Application

27

**The CCSS-M require a balance of:**

- Solid conceptual understanding
- Procedural skill and fluency
- Application of skills in problem solving situations

**Pursuit of all three requires equal intensity in time, activities, and resources.**

# Required Fluencies in K-6

28

Grade	Required Fluency
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 <sup>1</sup> Add/subtract within 100 (pencil and paper)
3	Multiply/divide within 100 <sup>2</sup> Add/subtract within 1000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$ , $p(x + q) = r$
8	Solve simple $2 \times 2$ systems by inspection

# Building Student Mathematical Understanding and Procedural Fluency

29

What happens when a student learns a procedure before Understanding?



# Cognitively Guided Instruction



## Guiding principle:

- Children bring intuitive knowledge of mathematics to problem solving situations

## Teaching approach:

- Elicit children's mathematical thinking and use it as the basis for instructional decisions
- Connect new mathematics ideas to prior knowledge to promote learning math with conceptual understanding

## Overarching goal:

- Foster students' development as problem solvers who bring skill and confidence to mathematics situations
- Develop "mathematically proficient" students

# What is CGI?



Cognitively Guided Instruction (CGI) is a professional development program based on an integrated program of research focused on

the development of students' mathematical thinking;  
instruction that influences that development;  
teachers knowledge and beliefs that influence their instructional practices; and  
the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking

How might we prepare our students for their future success in a global economy?

